

Pre-Algebra - Order of Operations

Objective: Evaluate expressions using the order of operations, including the use of absolute value.

When simplifying expressions it is important that we simplify them in the correct order. Consider the following problem done two different ways:

Example 1.

$\underline{2+5} \cdot 3$	Add First	$2 + \underline{5 \cdot 3}$	Multiply
$\underline{7 \cdot 3}$	Multiply	$\underline{2+15}$	Add
21	Solution	17	Solution

The previous example illustrates that if the same problem is done two different ways we will arrive at two different solutions. However, only one method can be correct. It turns out the second method, 17, is the correct method. The order of

operations ends with the most basic of operations, addition (or subtraction). Before addition is completed we must do repeated addition or multiplication (or division). Before multiplication is completed we must do repeated multiplication or exponents. When we want to do something out of order and make it come first we will put it in parenthesis (or grouping symbols). This list then is our order of operations we will use to simplify expressions.

Order of Operations:

Parenthesis (Grouping)

Exponents

Multiply and Divide (Left to Right)

Add and Subtract (Left to Right)

Multiply and Divide are on the same level because they are the same operation (division is just multiplying by the reciprocal). This means they must be done left to right, so some problems we will divide first, others we will multiply first. The same is true for adding and subtracting (subtracting is just adding the opposite).

Often students use the word PEMDAS to remember the order of operations, as the first letter of each operation creates the word PEMDAS. However, it is the

author's suggestion to think about PEMDAS as a vertical word written as:

P
E
MD
AS

so we don't forget that multiplication and division are done left to right (same with addition and subtraction). Another way students remember the order of operations is to think of a phrase such as "Please Excuse My Dear Aunt Sally" where each word starts with the same letters as the order of operations start with.

World View Note: The first use of grouping symbols are found in 1646 in the Dutch mathematician, Franciscus van Schooten's text, Vieta. He used a bar over the expression that is to be evaluated first. So problems like $2(3 + 5)$ were written as $2 \cdot \overline{3 + 5}$.

Example 2.

$2 + 3(\underline{9 - 4})^2$	Parenthesis first
$2 + 3(\underline{5})^2$	Exponents
$2 + 3(\underline{25})$	Multiply
$\underline{2 + 75}$	Add

It is very important to remember to multiply and divide from from left to right!

Example 3.

$$\begin{array}{ll} \underline{30 \div 3} \cdot 2 & \text{Divide first (left to right!)} \\ \underline{10} \cdot 2 & \text{Multiply} \\ 20 & \text{Our Solution} \end{array}$$

In the previous example, if we had multiplied first, five would have been the answer which is incorrect.

If there are several parenthesis in a problem we will start with the inner most parenthesis and work our way out. Inside each parenthesis we simplify using the order of operations as well. To make it easier to know which parenthesis goes with which parenthesis, different types of parenthesis will be used such as $\{ \}$ and $[]$ and $()$, these parenthesis all mean the same thing, they are parenthesis and must be evaluated first.

Example 4.

$$\begin{array}{ll} 2\{8^2 - 7[32 - 4(3^2 + 1)](-1)\} & \text{Inner most parenthesis, exponents first} \\ 2\{8^2 - 7[32 - 4(9 + 1)](-1)\} & \text{Add inside those parenthesis} \\ 2\{8^2 - 7[32 - 4(10)](-1)\} & \text{Multiply inside inner most parenthesis} \\ 2\{8^2 - 7[\underline{32 - 40}](-1)\} & \text{Subtract inside those parenthesis} \\ 2\{8^2 - 7[-8](-1)\} & \text{Exponents next} \\ 2\{64 - 7[-8](-1)\} & \text{Multiply left to right, sign with the number} \\ 2\{64 + 56(-1)\} & \text{Finish multiplying} \\ 2\{\underline{64 - 56}\} & \text{Subtract inside parenthesis} \\ \underline{2\{8\}} & \text{Multiply} \\ 16 & \text{Our Solution} \end{array}$$

As the above example illustrates, it can take several steps to complete a problem. The key to successfully solve order of operations problems is to take the time to show your work and do one step at a time. This will reduce the chance of making a mistake along the way.

There are several types of grouping symbols that can be used besides parenthesis. One type is a fraction bar. If we have a fraction, the entire numerator and the entire denominator must be evaluated before we reduce the fraction. In these cases we can simplify in both the numerator and denominator at the same time.

Example 5.

$$\frac{\tilde{2}^4 - (-8) \cdot 3}{\underline{15 \div 5} - 1} \quad \text{Exponent in the numerator, divide in denominator}$$

$$\frac{16 - \overbrace{(-8) \cdot 3}}{\underbrace{3 - 1}} \quad \text{Multiply in the numerator, subtract in denominator}$$

$$\frac{\overbrace{16 - (-24)}}{2} \quad \text{Add the opposite to simplify numerator, denominator is done.}$$

$$\frac{40}{2} \quad \text{Reduce, divide}$$

20 Our Solution

Another type of grouping symbol that also has an operation with it, absolute value. When we have absolute value we will evaluate everything inside the absolute value, just as if it were a normal parenthesis. Then once the inside is completed we will take the absolute value, or distance from zero, to make the number positive.

Example 6.

$$\begin{array}{ll}
 1 + 3|-4^2 - (-8)| + 2|3 + (-5)^2| & \text{Evaluate absolute values first, exponents} \\
 1 + 3|\underbrace{-16 - (-8)}| + 2|3 + 25| & \text{Add inside absolute values} \\
 1 + 3|8| + 2|28| & \text{Evaluate absolute values} \\
 1 + 3(8) + 2(28) & \text{Multiply left to right} \\
 1 + 24 + 2(28) & \text{Finish multiplying} \\
 1 + 24 + 56 & \text{Add left to right} \\
 25 + 56 & \text{Add} \\
 81 & \text{Our Solution}
 \end{array}$$

The above example also illustrates an important point about exponents. Exponents only are considered to be on the number they are attached to. This means when we see -4^2 , only the 4 is squared, giving us $-(4^2)$ or -16 . But when the negative is in parentheses, such as $(-5)^2$ the negative is part of the number and is also squared giving us a positive solution, 25.



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0.3 Practice - Order of Operation

Solve.

1) $-6 \cdot 4(-1)$

3) $3 + (8) \div |4|$

5) $8 \div 4 \cdot 2$

7) $[-9 - (2 - 5)] \div (-6)$

9) $-6 + (-3 - 3)^2 \div |3|$

11) $4 - 2|3^2 - 16|$

13) $[-1 - (-5)]|3 + 2|$

15) $\frac{2 + 4|7 + 2^2|}{4 \cdot 2 + 5 \cdot 3}$

17) $[6 \cdot 2 + 2 - (-6)](-5 + \left| \frac{-18}{6} \right|)$

19) $\frac{-13 - 2}{2 - (-1)^3 + (-6) - [-1 - (-3)]}$

21) $6 \cdot \frac{-8 - 4 + (-4) - [-4 - (-3)]}{(4^2 + 3^2) \div 5}$

23) $\frac{2^3 + 4}{-18 - 6 + (-4) - [-5(-1)(-5)]}$

25) $\frac{5 + 3^2 - 24 \div 6 \cdot 2}{[5 + 3(2^2 - 5)] + |2^2 - 5|^2}$

2) $(-6 \div 6)^3$

4) $5(-5 + 6) \cdot 6^2$

6) $7 - 5 + 6$

8) $(-2 \cdot 2^3 \cdot 2) \div (-4)$

10) $(-7 - 5) \div [-2 - 2 - (-6)]$

12) $\frac{-10 - 6}{(-2)^2} - 5$

14) $-3 - \{3 - [-3(2 + 4) - (-2)]\}$

16) $-4 - [2 + 4(-6) - 4 - |2^2 - 5 \cdot 2|]$

18) $2 \cdot (-3) + 3 - 6[-2 - (-1 - 3)]$

20) $\frac{-5^2 + (-5)^2}{|4^2 - 2^5| - 2 \cdot 3}$

22) $\frac{-9 \cdot 2 - (3 - 6)}{1 - (-2 + 1) - (-3)}$

24) $\frac{13 + (-3)^2 + 4(-3) + 1 - [-10 - (-6)]}{\{[4 + 5] \div [4^2 - 3^2(4 - 3) - 8]\} + 12}$



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Answers - Order of Operation

- | | | |
|---------|-----------|-----------|
| 1) 24 | 10) -6 | 19) 3 |
| 2) -1 | 11) -10 | 20) 0 |
| 3) 5 | 12) -9 | 21) -18 |
| 4) 180 | 13) 20 | 22) -3 |
| 5) 4 | 14) -22 | 23) -4 |
| 6) 8 | 15) 2 | 24) 3 |
| 7) 1 | 16) 28 | 25) 2 |
| 8) 8 | 17) -40 | |
| 9) 6 | 18) -15 | |



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